Assignment I

Ronan Kenny
Tommy Link
Guus Bouwens
Xavier Raap

March 6, 2023

Data

The dataset contains quarterly French data on the gross domestic product (GDP), consumer price index (CPI), and interest rate (IR), a monetary policy interest rate that is set by the central bank. The data of the GDP and CPI are already in growth rates, hence their data can be interpreted as the GDP growth and the inflation rate, respectively. In the following sections, we will analyze these three time series by setting up models to compute unknown coefficients, comparing performance measures, and performing hypothesis tests.

Part A

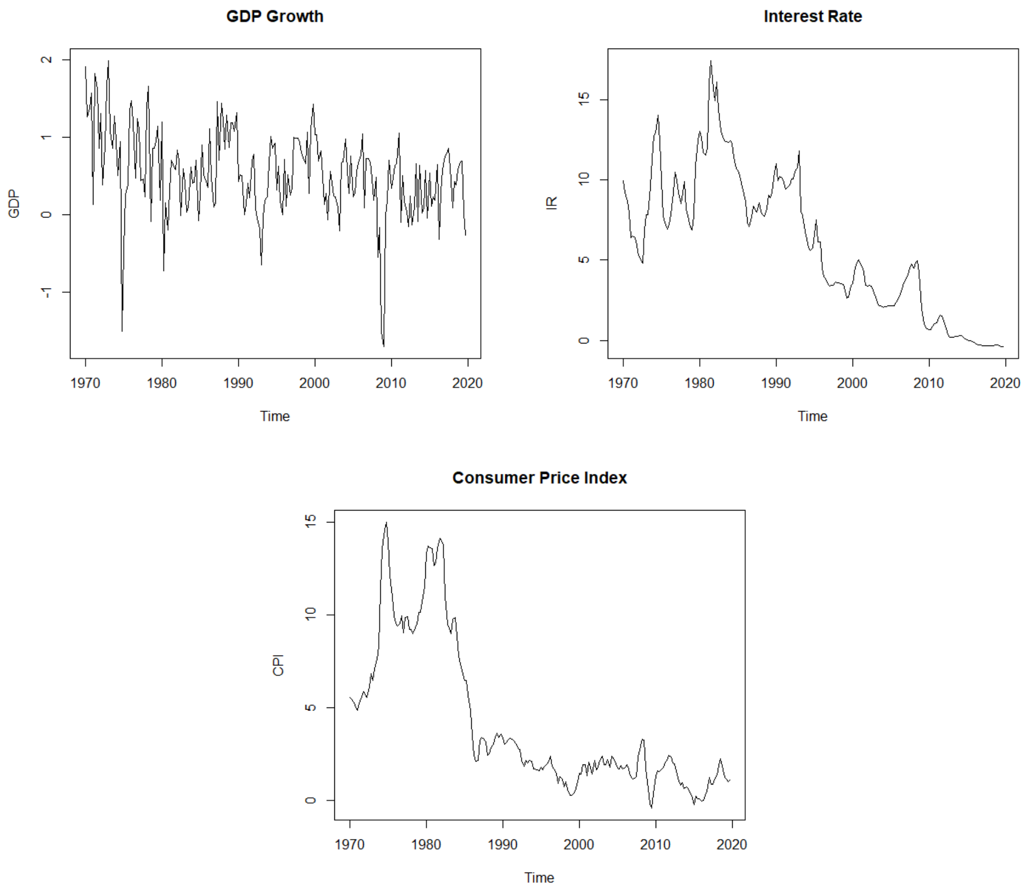

Figure 1: Time series plots of three economic indicators of the French economy

An interesting feature from these plots include the fact that both the interest rate and CPI have steadily declined since the year 1982.

Part B

We now turn to the estimation of our multiple time series. Let $\mathbf{y_t} := \begin{bmatrix} GDP_t \\ CPI_t \end{bmatrix}$. Consider the vector autoregressive (VAR) model with 3 lags for the GDP growth and inflation rate, i.e. a bivariate $VAR(3)$:

$$\mathbf{y_t} = \mathbf{c} + \mathbf{A_1 y_{t-1}} + \mathbf{A_2 y_{t-2}} + \mathbf{A_3 y_{t-3}} + \mathbf{u_t}, \qquad \mathbf{u_t} \sim \mathbf{WN(0, \Sigma_u)} \tag{1}$$

The unknown quantities in (1) are $\mathbf{c}$, $\mathbf{A_1}$, $\mathbf{A_2}$, $\mathbf{A_3}$ and $\mathbf{\Sigma_u}$. By making use of our R code and using the regression equation $\mathbf{Y} = \mathbf{BZ} + \mathbf{U}$ (details can be found on slide 26 of week 2), we find the following results for the estimation of the unknown quantities:

$$\hat{\mathbf{B}} = \begin{bmatrix} 0.180 & 0.335 & 0.006 & 0.189 & -0.148 & 0.091 & 0.145 \\ -0.082 & 0.159 & 1.438 & 0.074 & -0.488 & 0.043 & 0.032 \end{bmatrix}.$$

Hence $\hat{\mathbf{c}} = \begin{bmatrix} 0.180 \\ -0.082 \end{bmatrix}$, $\widehat{\mathbf{A_1}} = \begin{bmatrix} 0.335 & 0.006 \\ 0.159 & 1.438 \end{bmatrix}$, $\widehat{\mathbf{A_2}} = \begin{bmatrix} 0.189 & -0.148 \\ 0.074 & -0.488 \end{bmatrix}$ and

$\widehat{\mathbf{A_3}} = \begin{bmatrix} 0.091 & 0.145 \\ 0.043 & 0.032 \end{bmatrix}$. Furthermore we obtain that $\widehat{\mathbf{\Sigma_u}} = \begin{bmatrix} 0.222 & 0.016 \\ 0.016 & 0.260 \end{bmatrix}$.

When writing our model into companion form (details can be found on slide 8 of week 2), we find that the eigenvalues of $\widehat{\mathbf{A}}$ are $\boldsymbol{\lambda} = \begin{bmatrix} 0.976 & 0.636 & 0.636 & 0.340 & 0.340 & 0.072 \end{bmatrix}'$. It is evident that $|\lambda_j(\widehat{\mathbf{A}})| < 1 \ \forall j = 1, 2, \ldots, 6$. Therefore, the stability criterion is fulfilled and we conclude that our bivariate $VAR(3)$ model is stable.

Part C

For this part, we will compute and compare the Akaike Information Criterion (AIC), Schwartz Criterion (SC), and Hannan-Quinn (HQ) information criterion for different $VAR(p)$ models. Given that we have computed $\widehat{\boldsymbol{\Sigma}_\mathbf{u}}^{\mathbf{ML}}$ for a certain $VAR(p)$ model, the information criteria can be calculated as follows:

- $AIC(p) = \ln(\det(\widehat{\boldsymbol{\Sigma}_\mathbf{u}}^{\mathbf{ML}})) + \frac{2}{T}(pK^2 + K)$
- $HQ(p) = \ln(\det(\widehat{\boldsymbol{\Sigma}_\mathbf{u}}^{\mathbf{ML}})) + \frac{2\ln(\ln(T))}{T}(pK^2 + K)$
- $SC(p) = \ln(\det(\widehat{\boldsymbol{\Sigma}_\mathbf{u}}^{\mathbf{ML}})) + \frac{\ln(T)}{T}(pK^2 + K)$

The results are summarized in Table 1.

Information criterion \ Lag order p	1	2	3
AIC	-2.463	-2.712	-2.722
HQ	-2.423	-2.655	-2.618
SC	-2.363	-2.552	-2.480

Table 1: Performance measures per model

We are interested in finding the lag order $\hat{p} = \arg\min_p IC(p)$, where $IC(p)$ is any of the aforementioned information criteria with lag order p. In finite samples with $T \geq 16$ it holds that $AIC(\hat{p}) \geq HQ(\hat{p}) \geq SC(\hat{p})$. The SC therefore often chooses the smallest lag order, making it the most conservative criterion. Similarly, the AIC often chooses the largest lag order, making it the least conservative criterion. From Table 1 we observe that the lag order that minimizes the AIC is $p = 3$ with a corresponding value of -2.037. For that reason, the $VAR(3)$ model is the best choice to describe the data according to the least conservative information criterion.

Part D

As mentioned above, the VAR model that best describes the data according to the least conservative information criterion AIC is the VAR(3) model. Figure 2 shows the four plots of the response functions of our system after impulses of one unit, while Figure 3 shows the four associated plots of the accumulated impulse response functions. The impulse responses are mapped on a 10 period horizon.

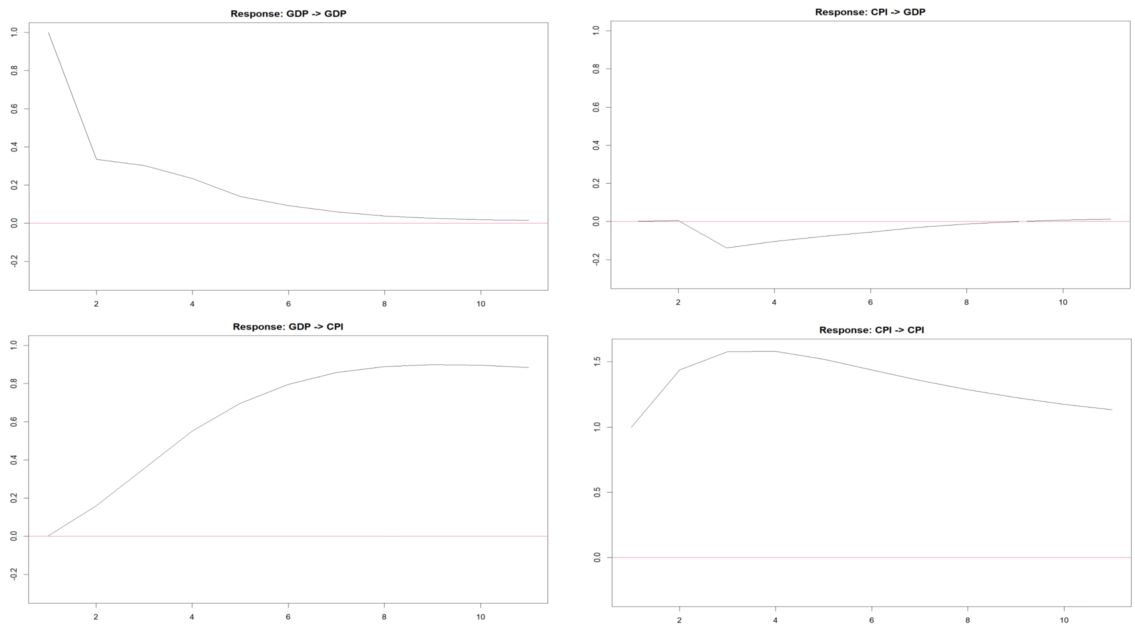

Figure 2: Response functions after impulses of one unit

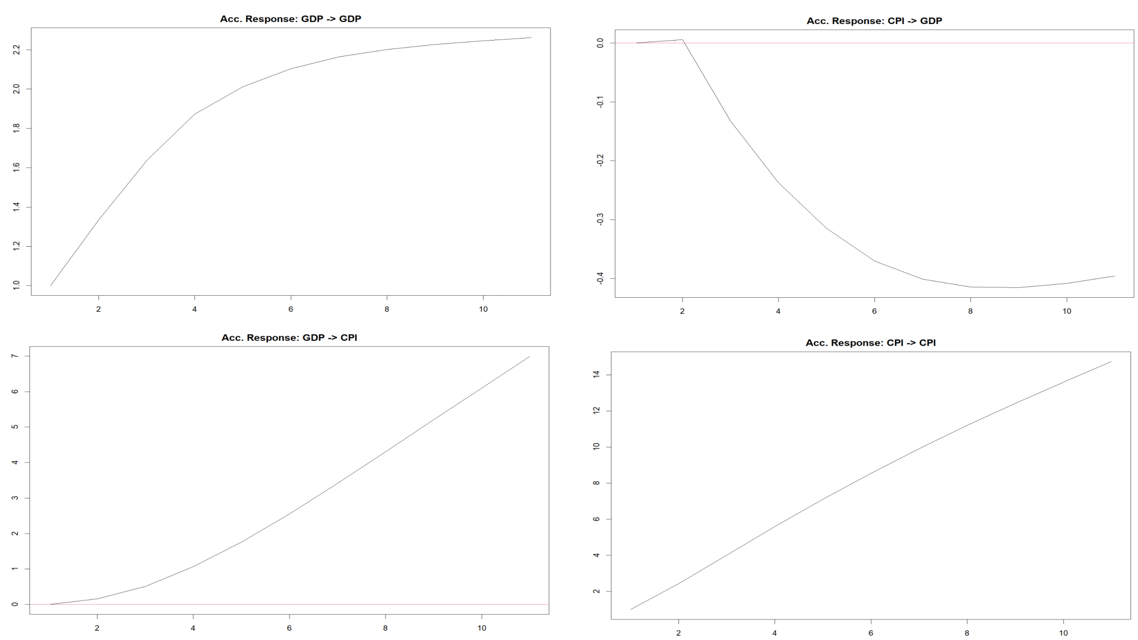

Figure 3: Accumulated response functions after impulses of one unit

There is much to be said about the impulse response functions in 2. It is important to note that the graphs are scaled differently on the y-axis, so this must be taken into consideration when analyzing the effects of the unit shocks. We see that the CPI has quite a minimal negative impact on the GDP for the first 3 periods, before steadily having an positive effect and leveling off to zero. On the other hand, the GDP has a positive influence on the CPI for the first 7 or 8 periods before the effect begins to decrease back towards zero. Although it might not be apparent from the figure due to the horizon only being 10 periods, if the horizon is set to a larger horizon (e.g. 100), the decrease towards zero is much clearer to see. Therefore, we see that due to the stability of the systems, the effect of the unit shock dies off after a certain point (different for each system), and levels off to zero. The accumulated response functions in 3 show the overall effect over the 10 period horizon. As time goes on, we see that the CPI negatively affects the GDP for the first 8 periods before marginally beginning to increase to a positive effect. The GDP has a positive impact on the CPI for the 10 year period in the figure, but as mentioned previously, if we had visualized a longer horizon, we would expect to see this accumulated effect level out at some point.

Part E

In a similar fashion, as we have seen in part B, we define $\mathbf{y_t} := \begin{bmatrix} GDP_t \\ IR_t \\ CPI_t \end{bmatrix}$ and consider the $VAR(2)$ model for the GDP growth, interest rate, and inflation rate:

$$\mathbf{y_t} = \mathbf{c} + \mathbf{A_1}\mathbf{y_{t-1}} + \mathbf{A_2}\mathbf{y_{t-2}} + \mathbf{u_t}, \qquad \mathbf{u_t} \sim \mathbf{WN}(\mathbf{0}, \mathbf{\Sigma_u}) \tag{2}$$

We find the following results for the estimation of the unknown quantities:

$$\hat{\mathbf{B}} = \begin{bmatrix} 0.232 & 0.361 & 0.007 & -0.060 & 0.220 & -0.017 & 0.069 \\ -0.082 & 0.268 & 1.236 & 0.034 & 0.099 & -0.288 & 0.003 \\ -0.036 & 0.135 & 0.107 & 1.381 & 0.065 & -0.107 & -0.400 \end{bmatrix}.$$

Hence $\hat{\mathbf{c}} = \begin{bmatrix} 0.232 \\ -0.082 \\ -0.036 \end{bmatrix}$, $\widehat{\mathbf{A_1}} = \begin{bmatrix} 0.361 & 0.007 & -0.060 \\ 0.268 & 1.236 & 0.034 \\ 0.135 & 0.107 & 1.381 \end{bmatrix}$, $\widehat{\mathbf{A_2}} = \begin{bmatrix} 0.220 & -0.017 & 0.069 \\ 0.099 & -0.288 & 0.003 \\ 0.065 & -0.107 & -0.400 \end{bmatrix}$. Furthermore we obtain that $\widehat{\mathbf{\Sigma_u}} = \begin{bmatrix} 0.227 & 0.073 & 0.017 \\ 0.073 & 0.532 & 0.112 \\ 0.017 & 0.112 & 0.254 \end{bmatrix}$.

Part F

The final section concerns itself with the question of whether or not the interest rate Granger-causes the GDP growth and inflation rate as a group. Define $z_t = IR_t$ and $x_t = \begin{bmatrix} GDP_t \\ CPI_t \end{bmatrix}$. We now wish to test if $z_t \xrightarrow{Gr} x_t$. Writing out the three-dimensional $VAR(2)$ model from part E:

$$\begin{bmatrix} GDP_t \\ CPI_t \\ IR_t \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + \begin{bmatrix} a_{11,1} & a_{12,1} & a_{13,1} \\ a_{21,1} & a_{22,1} & a_{23,1} \\ a_{31,1} & a_{32,1} & a_{33,1} \end{bmatrix} \begin{bmatrix} GDP_{t-1} \\ CPI_{t-1} \\ IR_{t-1} \end{bmatrix} + \begin{bmatrix} a_{11,2} & a_{12,2} & a_{13,2} \\ a_{21,2} & a_{22,2} & a_{23,2} \\ a_{31,2} & a_{32,2} & a_{33,2} \end{bmatrix} \begin{bmatrix} GDP_{t-2} \\ CPI_{t-2} \\ IR_{t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix} \tag{3}$$

The test boils down to testing whether the coefficients $a_{13,1}, a_{23,1}, a_{13,2}, a_{23,2}$ are all simultaneously equal to zero. Our hypothesis test is as follows: $H_0 = C\tilde{\mathbf{B}} = 0$ (no Granger causality) vs $H_1 = C\tilde{\mathbf{B}} \neq 0$ (Granger causality), $\tilde{\mathbf{B}}$ being the vectorized version of the coefficient matrix $\mathbf{B} = (\mathbf{c} : \mathbf{A_1} : \mathbf{A_2})$. We assume a fixed significance level of $\alpha = 5\%$. If we define our matrix C to be

$$C = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0 \end{bmatrix} \tag{4}$$

then $C\tilde{\mathbf{B}} = \begin{bmatrix} a_{13,1} \\ a_{23,1} \\ a_{13,2} \\ a_{23,2} \end{bmatrix}$ and we can test our null hypothesis H_0. The test statistic that we use:

$$\lambda = \frac{1}{N}(C\tilde{\mathbf{B}})' \left[C((ZZ')^{-1} \otimes \widehat{\mathbf{\Sigma_u}}^{\mathbf{LS}})C' \right]^{-1} C\tilde{\mathbf{B}}$$

Where $\widehat{\mathbf{\Sigma_u}}^{\mathbf{LS}} = \frac{1}{T-Kp-1} \sum_{t=1}^{T} \hat{\mathbf{u}}_\mathbf{t} \hat{\mathbf{u}}_\mathbf{t}'$ and $N = 4$. Under H_0, we have that $\lambda \approx F_{N, KT-K^2p-K}$. Using the $VAR(3)$ model that we found in part C, we find a value of 1.24 for the test statistic with a corresponding p-value of 0.293. We thus fail to reject H_0 at significance level $\alpha = 5\%$ and are not able to conclude whether or not the interest rate Granger-causes the GDP growth and inflation rate as a group.

Appendix

Reading in data

```
rm(list=ls())
library(readr)
data <- read.csv("data_assignment1_2023.csv")
```

Code for part A

```
# defines the individual data columns as there own entity
gdp.data <- data[3]
ir.data <- data[4]
cpi.data <- data[5]

# plots the individual graphs
plot(ts(gdp.data, frequency = 4, start=c(1970,2)), main="Quarterly French Gross Domestic Product (GDP
plot(ts(ir.data, frequency = 4, start=c(1970,2)), main="Quarterly French Interest Rate (IR)", ylab =
plot(ts(cpi.data, frequency = 4, start=c(1970,2)), main="Quarterly French Consumer Price Index (CPI)"

# defines data that will be used in parts B - D
gdp.ir.cpi.data <- data[c(3,5)]
ts.values <- ts(gdp.ir.cpi.data, frequency=4, start=c(1970,2))
```

Code for part B

```
  # function for estimating VAR(p) models. Input the data and the lag order p
  EstimateVAR <- function(gdp.ir.cpi.data,order){

    gdp.ir.cpi.data.mat <- as.matrix(gdp.ir.cpi.data) #data is converted into matrix
    T.orig <- nrow(gdp.ir.cpi.data.mat) # the number of values in the time-series
    T.fin <- T.orig - order # the number of values minus the lag order
    start.point = order + 1 # the point where the dependent variable will begin
    Y.mat <- t(gdp.ir.cpi.data.mat[start.point:T.orig,]) # dependent variables matrix

  # The following if-statements are very similar but have small changes based on their order
    if (order == 3) {
        # creates matrix of regressors
        Z.mat <- t(cbind(rep(1, T.fin),
            gdp.ir.cpi.data.mat[3:(T.orig-1),],
            gdp.ir.cpi.data.mat[2:(T.orig-2),],
            gdp.ir.cpi.data.mat[1:(T.orig-3),]))

        # estimated coefficient matrix using OLS
        B.hat <- Y.mat %*% t(Z.mat) %*% solve(Z.mat %*% t(Z.mat))

        # companion form coefficient matrix
        dim.VAR <- nrow(Y.mat)

        # zero matrix
        zero.dim.VAR <- matrix(rep(0,dim.VAR^2), ncol=dim.VAR)

        # coefficient matrices
```

```
        A.hat <- rbind(B.hat[,2:ncol(B.hat)],
            cbind(diag(dim.VAR), zero.dim.VAR,zero.dim.VAR),
            cbind(zero.dim.VAR, diag(dim.VAR), zero.dim.VAR))

        # eigenvalues from characteristic polynomial
        print(sort(abs(eigen(A.hat)$values), decreasing = TRUE))
    }

    if (order == 2) {
        Z.mat <- t(cbind(rep(1, T.fin),
            gdp.ir.cpi.data.mat[2:(T.orig-1),],
            gdp.ir.cpi.data.mat[1:(T.orig-2),]))

        B.hat <- Y.mat %*% t(Z.mat) %*% solve(Z.mat %*% t(Z.mat))

        dim.VAR <- nrow(Y.mat)
        zero.dim.VAR <- matrix(rep(0,dim.VAR^2), ncol=dim.VAR)

        A.hat <- rbind(B.hat[,2:ncol(B.hat)],
            cbind(diag(dim.VAR), zero.dim.VAR))

        sort(abs(eigen(A.hat)$values), decreasing = TRUE)

        print(sort(abs(eigen(A.hat)$values), decreasing = TRUE))
    }

    if (order == 1) {
        Z.mat <- t(cbind(rep(1, T.fin),
            gdp.ir.cpi.data.mat[1:(T.orig-1),]))

        B.hat <- Y.mat %*% t(Z.mat) %*% solve(Z.mat %*% t(Z.mat))

        dim.VAR <- nrow(Y.mat)
        zero.dim.VAR <- matrix(rep(0,dim.VAR^2), ncol=dim.VAR)

        A.hat <- rbind(B.hat[,2:ncol(B.hat)])

        print(sort(abs(eigen(A.hat)$values), decreasing = TRUE))
    }
    return(list("Y.mat" = Y.mat, "Z.mat" = Z.mat, "B.hat" = B.hat,"T.fin" = T.fin,"order" = order, "d
}
```

Code for part C

```
# Code is very similar for all of the VAR(p) models, however varies due to the lag order a small bit
### VAR(3) ###
values <- EstimateVAR(gdp.ir.cpi.data,3) #read in the return values from the VAR(p) function

# matrix for the estimated least squares residuals
U.hat.mat <- values$Y.mat - values$B.hat %*% values$Z.mat
sample.size <- 197 # sample size 197 for all VAR(p) in order to make reasonable comparision, i.e. T.f
order.VAR <- values$order # lag order of the VAR(p)

# Estimating the covariance matrix
Sigma.u.hat.ML <- 1/values$T.fin*crossprod(t(U.hat.mat))
det.Sigma.u.hat <- det(Sigma.u.hat.ML)
```

```
# Information Criteria
AIC.VAR3 <-log(det.Sigma.u.hat) + (2/sample.size) *
(order.VAR*values$dim.VAR^2+values$dim.VAR)

HQ.VAR3 <-log(det.Sigma.u.hat) +
(2*log(log(sample.size)))/sample.size *
(values$dim.VAR^2*order.VAR+values$dim.VAR)

SC.VAR3 <-log(det.Sigma.u.hat) +
log(sample.size)/sample.size *
(values$dim.VAR^2*order.VAR+values$dim.VAR)
values$dim.VAR

### VAR(2) ###
values <- EstimateVAR(gdp.ir.cpi.data,2)
U.hat.mat <- values$Y.mat - values$B.hat %*% values$Z.mat
U.hat.mat <- U.hat.mat[,-c(198)] #remove this to be the same length as T.fin and sample size which ac
sample.size <- 197
values$T.fin <- 197
order.VAR <- values$order

Sigma.u.hat.ML <- 1/values$T.fin*crossprod(t(U.hat.mat))
det.Sigma.u.hat <- det(Sigma.u.hat.ML)

AIC.VAR2 <-log(det.Sigma.u.hat) + (2/sample.size) *
(order.VAR*values$dim.VAR^2+values$dim.VAR)

HQ.VAR2 <-log(det.Sigma.u.hat) +
(2*log(log(sample.size)))/sample.size *
(values$dim.VAR^2*order.VAR+values$dim.VAR)

SC.VAR2 <-log(det.Sigma.u.hat) +
log(sample.size)/sample.size *
(values$dim.VAR^2*order.VAR+values$dim.VAR)

### VAR(1) ###
values <- EstimateVAR(gdp.ir.cpi.data,1)
U.hat.mat <- values$Y.mat - values$B.hat %*% values$Z.mat
U.hat.mat <- U.hat.mat[,-c(198,199)] #remove these to be the same length as T.fin and sample size whi
sample.size <- 197
values$T.fin <- 197
order.VAR <- values$order

Sigma.u.hat.ML <- 1/values$T.fin*crossprod(t(U.hat.mat))
det.Sigma.u.hat <- det(Sigma.u.hat.ML)

AIC.VAR1 <-log(det.Sigma.u.hat) + (2/sample.size) *
(order.VAR*values$dim.VAR^2+values$dim.VAR)

HQ.VAR1 <-log(det.Sigma.u.hat) +
(2*log(log(sample.size)))/sample.size *
(values$dim.VAR^2*order.VAR+values$dim.VAR)

SC.VAR1 <-log(det.Sigma.u.hat) +
log(sample.size)/sample.size *
(values$dim.VAR^2*order.VAR+values$dim.VAR)
```

```
# put results into a table
column1 <- c(AIC.VAR1, AIC.VAR2, AIC.VAR3)
column2 <- c(HQ.VAR1, HQ.VAR2, HQ.VAR3)
column3 <- c(SC.VAR1, SC.VAR2, SC.VAR3)
results.c <- data.frame(column1, column2, column3)
colnames(results.c) <- c('AIC','HQ','SC')
rownames(results.c)<-  c('1','2','3')
t(results.c)
```

Code for part D

```
values <- EstimateVAR(gdp.ir.cpi.data,3) # return values from estimated VAR(p) model

horizon <- 10 # the number of periods we would like to use

# building the coefficient matrix
A.hat.array <- array(0, dim=c(2,2,horizon))
A.hat.array[,,1:3] = array(values$B.hat[,-1], dim=c(2,2,3))
A.hat.array

# building the Impulse Responses array using recursion
IR.array <- array(NA, dim=c(2,2,(horizon+1)))
IR.array[,,1] = diag(2)
for(i in 2:(horizon+1)){
  IR.here <- matrix(0, nrow=2, ncol=2)
  for(j in 1:(i-1)){
  IR.here <- IR.here + (IR.array[,,(i-j)] %*% A.hat.array[,,j])
  }
  IR.array[,,i]= IR.here
}

# putting the array into tabular form
IR.table <- cbind(IR.array[1,1,], IR.array[1,2,],
IR.array[2,1,], IR.array[2,2,])
colnames(IR.table) = c(
"Response: GDP -> GDP",
"Response: CPI -> GDP",
"Response: GDP -> CPI",
"Response: CPI -> CPI")

# plots of impulse response functions
plot(IR.table[,1], type="l", main=colnames(IR.table)[1],
ylim=c(-0.3,1), ylab="", xlab="")
abline(h=0, col=2)
plot(IR.table[,2], type="l", main=colnames(IR.table)[2],
ylim=c(-0.3,1), ylab="", xlab="")
abline(h=0, col=2)
plot(IR.table[,3], type="l", main=colnames(IR.table)[3],
ylim=c(-0.3,1), ylab="", xlab="")
abline(h=0, col=2)
plot(IR.table[,4], type="l", main=colnames(IR.table)[4],
ylim=c(-0.3,1.6), ylab="", xlab="")
abline(h=0, col=2)

### ACCUMULATED IRF ###
# building accumulated impulse response arrays
```

```
acc.IR.array <- array(NA, dim=c(2,2,(horizon+1)))
acc.IR.array[,,1] = IR.array[,,1]
for(i in 2:(horizon+1)){
  acc.IR.array[,,i] = apply(IR.array[,,1:i], c(1,2), sum)
}
# putting the accumulated impulse response arrays into tabular form
acc.IR.table <- cbind(acc.IR.array[1,1,], acc.IR.array[1,2,],
acc.IR.array[2,1,], acc.IR.array[2,2,])
colnames(acc.IR.table) = c(
  "Acc. Response: GDP -> GDP",
  "Acc. Response: CPI -> GDP",
  "Acc. Response: GDP -> CPI",
  "Acc. Response: CPI -> CPI")

# plots
plot(acc.IR.table[,1], type="l", main=colnames(acc.IR.table)[1],
  ylab="", xlab="")
abline(h=0, col=2)
plot(acc.IR.table[,2], type="l", main=colnames(acc.IR.table)[2],
  ylab="", xlab="")
abline(h=0, col=2)
plot(acc.IR.table[,3], type="l", main=colnames(acc.IR.table)[3],
  ylab="", xlab="")
abline(h=0, col=2)
plot(acc.IR.table[,4], type="l", main=colnames(acc.IR.table)[4],
  ylab="", xlab="")
abline(h=0, col=2)
```

Code for part E

```
# reading in data with all three gdp, cpi, ir
all.gdp.ir.cpi.data <- data[3:5]

values <- EstimateVAR(all.gdp.ir.cpi.data,2) # returning the values from the estimation

# calculating the covariance matrix similar as in part c
U.hat.mat <- values$Y.mat - values$B.hat %*% values$Z.mat
sample.size <- values$T.fin
order.VAR <- values$order
Sigma.u.hat.ML <- 1/values$T.fin*crossprod(t(U.hat.mat))
```

Code for part F

```
C.mat.test <- rbind(
  c(rep(0,9),1,rep(0,11)),
  c(rep(0,10),1,rep(0,10)),
  c(rep(0,18),1,rep(0,2)),
  c(rep(0,19),1,rep(0,1)))
nr.restr <- nrow(C.mat.test)
dim.VAR <- 3
vec.B.hat <- vec(values$B.hat)

Sigma.u.hat.OLS <- 1/(T.fin-order.VAR*dim.VAR-1)*crossprod(t(U.hat.mat))

covariance.mat1 <- C.mat.test %*% (solve(Z.mat %*% t(Z.mat)) %x%
Sigma.u.hat.OLS) %*% t(C.mat.test)
```

```
wald.test.stat1 <- (t(C.mat.test %*%vec.B.hat) %*%
solve(covariance.mat1) %*%
(C.mat.test %*%vec.B.hat))/nr.restr

1-pf(wald.test.stat1, df1=nr.restr, df2=(dim.VAR*values$T.fin-dim.VAR^2*order.VAR-dim.VAR))
```